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The influence of short term variations in AM CVn systems on LISA measurements

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ABSTRACT

We study the effect of short term variations of the evolution of AM CVn systems on their gravitational wave emissions and in particular LISA observations. We model the systems according to their equilibrium mass-transfer evolution as driven by gravitational wave emission and tidal interaction, and determine their reaction to a sudden perturbation of the system. This is inspired by the suggestion to explain the orbital period evolution of the ultra-compact binary systems V407 Vul and RX-J0806+1527 by non-equilibrium mass transfer. The characteristics of the emitted gravitational wave signal are deduced from a Taylor expansion of a Newtonian quadrupolar emission model, and the changes in signal structure as visible to the LISA mission are determined. We show that short term variations can significantly change the higher order terms in the expansion, and thus lead to spurious (non) detection of frequency derivatives. This may hamper the estimation of the parameters of the system, in particular their masses and distances. However, we find that overall detection is still secured as signals still can be described by general templates. We conclude that a better modelling of the effects of short term variations is needed to prepare the community for astrophysical evaluations of real gravitational wave data of AM CVn systems.

Key words: double white dwarfs – gravitational waves – methods: data analysis.

1 INTRODUCTION

The Laser Interferometer Space Antenna (LISA) is an ESA/NASA space-based gravitational wave (GW) laser interferometer designed to observe a wide range of sources in the frequency range $\sim 10^{-5}$ - 1 Hz (Bender 1998). At the moment LISA is in its design stage with an open launch window of 2018+. One of the important classes of objects are Galactic compact object binary systems, in particular double white dwarf binaries (Evans et al. 1987; Lipunov & Postnov 1988; Hils et al. 1990). LISA will identify several thousand new binaries – with the key feature of not being biased against short orbital periods – and will therefore contribute in a significant way to unveil the complex physical interactions in compact object binaries (e.g. Nelemans et al. 2004; Stroeer et al. 2005; Benacquista et al. 2007). A particular class of objects are the mass-transferring AM CVn binaries (Nelemans 2005). In these systems the orbital evolution of the system is not only determined by gravitational wave emission (and possible tidal interaction), but also by the mass redistribution in the system. A number of the currently known AM CVn systems are expected to serve as verification sources for LISA (Stroeer & Vecchio 2006; Roelofs et al. 2007).

In an earlier paper (Stroeer et al. 2005) we showed that in order to determine the system parameters, either the exact evolution of the gravitational wave frequency up to second order in a series expansions needs to be determined, or complementary electro-magnetic observations are needed. Here we revisit this result, focusing on possible short term variations in the equilibrium mass-transfer rate, as observed in many interacting binaries, and as have been proposed as explanation for the observed period decrease in the ultra-compact binary systems V407 Vul and RX J0806+1527 (Marsh & Nelemans 2005; Barros et al. 2005, and references therein). In addition, especially early in the evolution of AM CVn systems, helium novae are expected (Bildsten et al. 2007), leading to a sudden perturbation of the system.

In Sect. 2 we describe the basic model, the way we perturb the system and the calculation of the resulting gravitational wave signal. In Sec. 3 we discuss the results for one fiducial AM CVn system after which we conclude in Sec. 4.

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2 MODEL

2.1 Binary evolution and perturbation

In this section we briefly review the key elements of the evolution of an AM CVn system, and the model adopted to characterize short period perturbations of the mass transfer rate. Our formalism is based on Marsh et al. (2004) to which we refer the reader for more details.

The progenitor stage of an AM CVn system is characterized by the loss of orbital angular momentum due to the emission of gravitational waves. The lost orbital angular momentum drives the system to shorter orbital periods and smaller orbital separations. If the orbital separation is sufficiently small for the radius of the secondary star to overfill its Roche lobe, mass is transferred onto the primary star (accretor) through Roche lobe overflow. The accretor responds by spinning up, removing angular momentum from the orbit at an even higher rate. However, tidal interaction may force the accretor into co-rotation with the orbit, effectively returning the lost angular momentum. For stable systems, redistribution of mass in the system allows the orbit to expand, even at decreasing angular momentum. The binary thus reaches a "turning point".

The binary under consideration is characterized by the masses and radii of the primary (more massive and accreting) and secondary (less massive and mass transferring) star, $M_{1,2}$ and $R_{1,2}$ respectively. The binary orbital separation a and its period P evolve due to the change of orbital angular momentum $J_{\rm o}$ according to (e.g. Marsh et al. 2004)

$$\dot{J}_{\rm o} = \dot{J}_{\rm GR} + \sqrt{G\,M_1\,R_{\rm h}}\,\dot{M}_2 + \frac{kM_1R_1^2}{\tau_{\rm S}}\left(\Omega_{\rm s} - \Omega_{\rm o}\right); \qquad (1)$$

the three terms on the right-hand side of the equation represent the loss of angular momentum due to gravitational wave radiation $\dot{J}_{\rm GR}$, mass loss at rate $\dot{M}_2$, and tidal coupling, respectively. The latter contribution is parametrized as a function of the synchronization time-scale $\tau_{\rm S}$, and $R_{\rm h}$ identifies the radius around the accretor with the same angular momentum as the transferred matter (Verbunt & Rappaport 1998). $\Omega_{\rm s}$ and $\Omega_{\rm o}$ are the angular frequencies of the accretor's spin and the orbit, respectively and $k \approx 0.2$ (Marsh et al. 2004).

We model the mass transfer rate $\dot{M}_2$ as an adiabatic response of the donor

$$\dot{M}_2 = -f(M_1, M_2, a, R_2)\Delta^3\,, \qquad (2)$$

where the function $f(M_1, M_2, a, R_2)$ is given by Eq. (10) of Marsh et al. (2004) (as based on results from Webbink (1977)). The quantity Δ is the Roche lobe overflow factor, defined as

$$\Delta = R_2 - R_{\rm L}\,, \qquad (3)$$

where $R_{\rm L}$ is the Roche lobe's radius and a function of a and M_2/M_1 only. We use a simple zero-temperature mass-radius relation as in Marsh et al. (2004), neglect the effects of a realistic donor structure, that generally gives rise to larger donor radii, i.e. lower mass-transfer rates and more complex mass transfer behaviour (e.g. Deloye & Taam 2006; Deloye et al. 2007).

Mass transfer spins up the accretor. We therefore see a coupling of mass transfer rates and the response of the accretor due to tidal locking. This coupling can be expressed in a coupled differential equation system of the evolution of Δ and the evolution of Ω_s

$$\frac{1}{2R_2}\frac{d\Delta}{dt} = -\frac{\dot{J}_{\rm GR}}{J_{\rm o}} - \frac{kM_1R_1^2}{\tau_{\rm S}J_{\rm o}}\left(\Omega_{\rm s} - \Omega_{\rm o}\right) + \frac{\dot{M}_2}{M_2}\times \left(1 + \frac{\zeta_2 - \zeta_{r_L}}{2} - q - \sqrt{(1+q)\frac{R_h}{a}}\right), \qquad (4)$$

$$\frac{d\Omega_s}{dt} = \left(\lambda\Omega_s - \frac{\sqrt{GM_1R_h}}{kR_1^2}\right)\frac{\dot{M}_2}{M_1} - \frac{(\Omega_{\rm s} - \Omega_{\rm o})}{\tau_{\rm S}}. \qquad (5)$$

In the previous equation $q = M_2/M_1$ is the mass ratio and the parameters ζ_2, ζ_{r_L} and λ are given, respectively, by

$$\zeta_2 = \frac{d\log R_2}{d\log M_2}\,, \qquad (6)$$

$$\zeta_{r_L} = \frac{d\log R_L/a}{d\log M_2}\,, \qquad (7)$$

$$\lambda = 1 + 2\frac{d\log R_1}{d\log M_1} + \frac{d\log k}{d\log M_1}\,. \qquad (8)$$

We started the modelling of our fiducial AM CVn system in the progenitor phase and followed its evolution through the "turning point" towards the stable AM CVn stage assuming strong tidal locking ($\tau_{\rm S} = 0.2yrs$ as favoured by Marsh et al. (2004), redbut note the discussion of the very large uncertainty in this value in their section 6). Masses for the primary and secondary star of our fiducial binary were defined in the progenitor phase with $M_1 = 0.4M_\odot$ and $M_2 = 0.2M_\odot$.

We induce a perturbation by suddenly offsetting Δ (through a change in separation a that in turn changes the Roche lobe R_L), which directly impacts the mass transfer rates and spin evolution of the accreting star (see Eqs. 2,4 and 5). As the mass transfer in the AM CVn systems is calculated implicitly from the system parameters this is the simplest self-consistent way of perturbing the system. The expected timescale $\tau_{\rm c}$ for returning to equilibrium after such a perturbation is (Marsh & Nelemans 2005)

$$\tau_{\rm c} = \frac{1}{3}\frac{\Delta}{\dot{\Delta}} \qquad (9)$$

We triggered the perturbation when the system reaches a gravitational wave frequency of 8 mHz in its stable AM CVn branch. We perform two kinds of perturbations. First we decrease the Roche lobe overflow factor, yielding reduced mass transfer rates, possible e.g. when nova explosions eject mass from the accreting star, widening the orbit. Second, we increase the Roche lobe overflow yielding increased mass transfer rates. Such an effect may be caused by suddenly expanding stellar atmospheres of the mass donor, or decrease in orbit if the expanding nova shell interacts with the secondary, removing orbital angular momentum. We offset Δ such as to yield five times larger and five times smaller mass transfer rates (see Eq. 2). Effectively we disturb the equilibrium rates of the AM CVn system which balance Eq. 4 and Eq. 5, and force the system to evolve back to its equilibrium by itself.

A factor five as perturbation for mass transfer rates is a quite modest change. The separation only needs to be changed by much less than one per cent to achieve such an offset. Further, we restrict this case study to a system at GW frequency of 8 mHz, since we refer to novae as source for the perturbation. The occurrence of a nova depends on the ignition of the accreted layer, which happens much easier and more often at high mass transfer rates (i.e. short periods) than later (Bildsten et al. 2007).

2.2 Gravitational wave signal

We model the GW that is emitted from such a system at the leading Newtonian quadrupole order, only keeping the + polarisation and working in the real domain. This yields for the GW strain in the solar reference frame

$$h_{GW}(t) = A(t)\cos(\Phi_{GW}(t)) = A(t)\cos(2\pi t f_{GW}(t)), \qquad (10)$$

with

$$A(t) = -\frac{1}{D}\frac{G^2}{c^4}\frac{2M_1(t)M_2(t)}{a(t)} \qquad (11)$$

the amplitude of the strain ($M_{1,2}(t)$ masses of the stars in a binary as affected by mass transfer, $D, a(t)$ distance to and varying separation of the system respectively, G, c Keplers constant and the velocity of light respectively, $\Phi_{GW}(t)$ the phase and f_{GW} the frequency of the gravitational wave). We calculate the period of the AM CVn according to

$$(P(t)/2\pi)^2 = a(t)^3/G(M_1(t)+M_2(t)) \qquad (12)$$

(Kepler's third law) to derive the gravitational wave frequency

$$f_{GW}(t) = 2/P(t) \qquad (13)$$

(leading order quadrupolar radiation as found in circular orbits, we drop the subscript GW hereafter).

We do neglect the LISA instrumental response function in this research, and thus remain in the solar reference frame through out. We do note that this does not have repercussions for the conclusions, as the response function does not change the GW frequency evolution (it merely modulates it). In the remainder of this paper we drop the explicit reference to the time dependence in the parameters.

We expanded the evolving phase of gravitational wave emission from the AM CVn system as a Taylor series up to second order to determine the influence of the perturbation to the GW signal.

$$\Phi_{GW}(t) = 2\pi f_0 t + \pi f_0^{(1)} t^2 + \frac{\pi}{3} f_0^{(2)} t^3 \qquad (14)$$

We defined with $f_0, f_0^{(1)}, f_0^{(2)}$ spin-down parameters for the system, here stating the gravitational wave frequency, its first and second derivative in time at the origin of the series, time nominally set to zero at an instant after the sudden offset of Δ. $\Phi_{GW}(t)$ is further the number of wave-cycles one expects to record at LISA within a given observation time. For this the running time t is replaced with the total observation time T. In this view, the three terms to $\Phi_{GW}(T)$ are contributing separately to the total number of wave-cycles.

3 RESULTS

In Fig. 1 we show the frequency evolution of the unperturbed system (solid line) and the perturbed system (dashed and dotted lines). We observed a change in drift of the frequency of gravitational wave emissions after a perturbation struck the system. Perturbations that lead to increased mass transfer rates forced the system to separate at a higher pace, leaving the gravitational wave frequency to decrease at a faster pace as compared to a secular evolving AM CVn at the same stage of evolution (compare Fig. 1 dashed line with the solid line). Perturbations that decreased the mass transfer rate, in contrast, lead to a short in-spiral of the system over 59 years (see Fig. 1 dotted line). The system experiences a new "turning point" and then evolves to lower frequencies again.

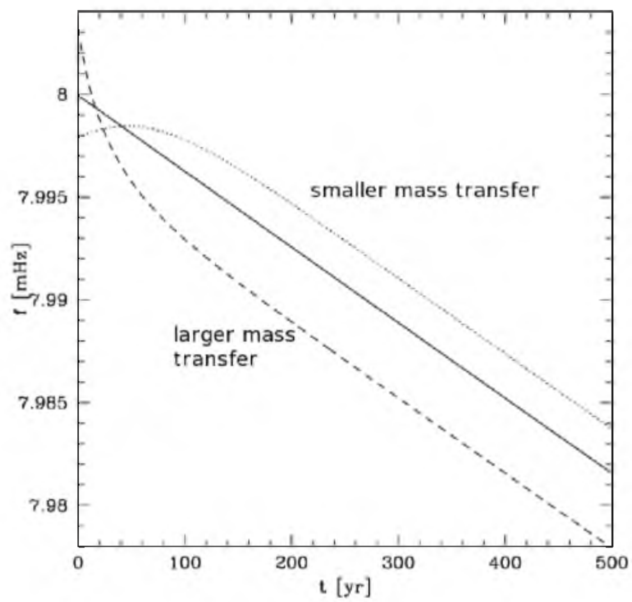

Figure 1. The evolution of the gravitational wave frequency of the fiducial AM CVn system over time, with the x axis counting from the beginning of the perturbation. The solid line denotes the equilibrium evolution, the dotted line an evolution with a mass transfer rate five times smaller than in equilibrium at onset of perturbation, the dashed line an evolution with a mass transfer rate five times bigger than in equilibrium at onset of perturbation.

We determined the total number of wave-cycles as detected during a one and five year mission of LISA and the contributions from the spin-down parameters to this total number by "starting" an observation at each individual year of the shown frequency evolution of the AM CVn in Fig. 1 (see Fig. 2 and Fig. 3). We found, that the shapes of the curves in Fig. 2 and Fig. 3 do not change between a one year observation and a five year observation, they simply scale according to T^2 and T^3 for the first and second derivative respectively. We therefore show explicit examples for a 5 year mission only.

The number of wave cycles contributed by $f_0^{(1)}$ clearly mirrors the altered frequency evolution. Perturbations that force increased mass transfer rates (dashed lines) yield faster separating systems, i.e. large negative contributions by $f_0^{(1)}$. We find the number of wave cycles from the first spin-down parameter to increase from -123 wave cycles to -990 wave cycles when integrating the number of wave cycles over a five year observation starting an instant after the sudden offset of Δ. Perturbations inducing decreased mass transfer rates (dotted lines) yield a short in-spiral of the system, we thus see first moderate positive then moderate negative con-

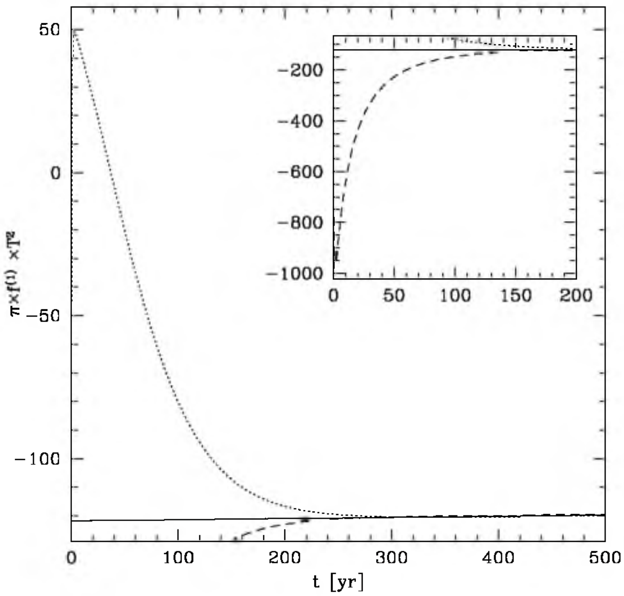

Figure 2. Number of wave-cycles as contributed by the first derivative of the gravitational wave frequency (spin-down parameter) over time as seen by LISA during a five year observation. Time as shown on the x axis refers to the timeline since perturbation struck the system, and a hypothetical LISA observation is started at every instance of this timeline to yield shown wave-cycle evolution. The solid line denotes the equilibrium evolution, the dotted line an evolution with a mass transfer rate five times smaller than in equilibrium at onset of perturbation, the dashed line an evolution with a mass transfer rate five times bigger than in equilibrium at onset of perturbation. The inlay shows a zoom in in order to cover the full spread of the evolution with increased mass transfer rate at onset. For details see text.

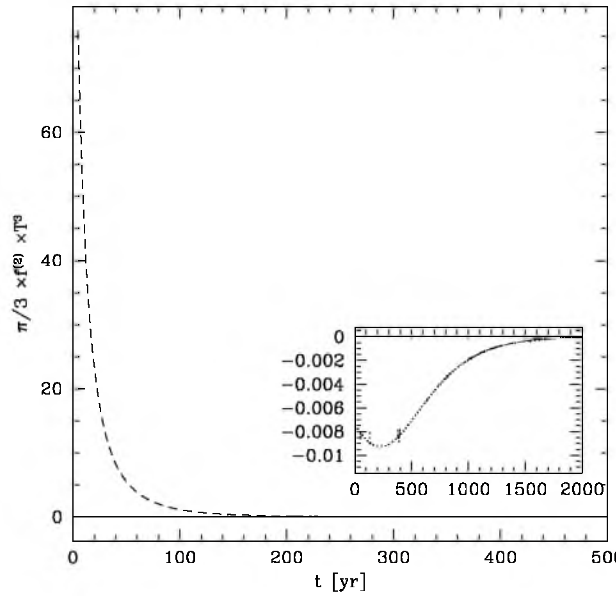

Figure 3. Number of wave-cycles as contributed by the second derivative of the gravitational wave frequency (spin-down parameter) over time as seen by LISA during a five year observation. Conventions as in Fig. 2

tributions by $f_0^{(1)}$ as separated by the new turning point in frequency. Here the significant alteration is more the reverse of the drift of the system, with +50 wave cycles immediately after the impulse instead of the quoted -123 wave cycles.

Interesting results can be found for $f_0^{(2)}$. This spin-down parameter normally does not contribute to the total number of wave-cycles for a system at 8mHz even in a five year observation. In case of a faster separating system however we suddenly see a significant increase in the contribution to the total number of wave-cycles, in fact by a factor of 10^6. Suddenly the second derivative becomes visible within a five year mission with +76 additional contributed wave-cycles, compared to a null contribution beforehand. In case of the system with decreased mass transfer we see an increase in the number of contributed wave cycles, but still the contribution is too small to be detected.

4 DISCUSSION AND CONCLUSIONS

We studied the reaction of the frequency evolution of a fiducial AM CVn system with a GW frequency of 8mHz, to sudden, but modest, perturbations of its orbital parameters. These perturbations are inspired by the observed and theoretically expected short term variations due to e.g. nova explosions. The system remains bound, changes its mass transfer rate, but on a time scale of hundreds of years, evolves back to its equilibrium mass transfer rate, as expected. However, if any of the thousands of individual binaries that LISA is expected to follow during its life time, is in such a perturbed state, the measured frequency evolution can be dramatically different than for the equilibrium case. The sign of the frequency derivative may change and the number of cycles contributed by the first and second derivative may increase by several tens to several hundreds in a 1 or 5 year mission life time. For template data analysis methods, the detection of these systems should still be secure, if sufficient spin-down parameters are used (up to second derivative, especially for a 5 year mission). However, care should be taken with deriving system parameters purely on the GW data, as the perturbations can give rise to wildly different derivatives, which, if interpreted as equilibrium evolution, would lead to extreme mass estimates, or for the systems with reversed evolution to the conclusion that no mass transfer is present. In addition in Stroeer et al. (2005) we concluded that $f_0^{(2)}$ is needed for distance estimations, but this works only when the system evolves in equilibrium.

It is difficult to estimate the number of LISA binaries that are affected by such short term variations. Many observed interacting binaries seem to have short term period evolution different from the secular evolution (e.g. Baptista et al. (2003)), suggesting it may be a quite common phenomenon. For the helium novae in AM CVn systems we can make a simple estimate. At the shortest periods, i.e. high mass transfer rates and for high accretor masses the amount of accumulated mass needed to ignite the nova is about 0.001 Msun (Bildsten et al. 2007). For a mass transfer rate of 10^{-5} Msun/yr, this leads to a nova every 100 years, influencing the evolution essentially all the time so we expect several LISA sources to be affected. However, most of the

LISA sources will have evolved to lower mass transfer rates, where the nova frequency is much lower and the chances of catching a system in the recovery phase are low.

We also tested repercussions of short term perturbations at an evolutionary stage of the fiducial AM CVn at which the orbital separation yields GW frequencies of order 3 mHz. This stage is set 3.5 million years after we found our AM CVn at 8mHz. Overall, tidal coupling is much less at such a separation so that the response to a shortterm perturbation is less dynamic. We find the timescale of the perturbation, τ_c, to be 691 yrs compared to 47.9 yrs. Therefore the maximum amount of increase in wavecycles, here found in the increased mass transferring scenario, is only a mere 2.5 wavecycles, and the second derivative is never visible in case of increased and decreased mass transfer perturbations at proposed scale. However, decreased mass transfer is still able to trigger a new turning point in the frequency evolution of the binary, the system inspirals again over a period of roughly 2000 years. We therefore conclude that although the magnitude of wavecycle increase is minor at 3 mHz the actual ability of the system to shortly inspiral is a reason of concern once again for parameter inference.

We thus re-enforce our earlier conclusion (Stroeer et al. 2005) that complementary electro-magnetic data from e.g. GAIA (Nelemans 2009) is an important ingredient for the disentangling of different processes that may contribute to the frequency evolution of compact binaries, such as mass transfer, tidal interaction and short term variability and the determination of accurate system parameters.

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